特征值与特征向量

练习

1. 求矩阵 A 的特征多项式：

a) $\begin{pmatrix} -2 & 1 \\ 1 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 5 \\ -5 & 1 \end{pmatrix}$

c) $\begin{pmatrix} -4 & -2 & 4 \\ -1 & -1 & 0 \\ 5 & 1 & -5 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 4 \\ -5 & 5 & -3 \\ 0 & 4 & 2 \end{pmatrix}$

e) $\begin{pmatrix} -3 & -4 & -3 \\ -1 & 4 & -3 \\ 3 & -2 & -5 \end{pmatrix}$ f) $\begin{pmatrix} 0 & -4 & -1 \\ 5 & -5 & -3 \\ 3 & -3 & 4 \end{pmatrix}$

2. 已知矩阵 A 的一个特征向量为 $\mathbf{v}$，求它所对应的特征值。其中 $A, \mathbf{v}$ 为

a) $v = \left(1, \frac{1}{3}\right)^T, A = \begin{pmatrix} -8 & 24 \\ -2 & 6 \end{pmatrix}$ b) $v = \left(1, \frac{1}{3}\right)^T, A = \begin{pmatrix} -6 & 24 \\ -2 & 8 \end{pmatrix}$

c) $v = \left(1, 0, -\frac{1}{4}\right)^T, A = \begin{pmatrix} -10 & -24 & -48 \\ 12 & 26 & 48 \\ -4 & -8 & -14 \end{pmatrix}$ d) $v = (1, 0, -1)^T, A = \begin{pmatrix} 10 & -32 & 8 \\ 2 & -6 & 2 \\ -2 & 8 & 0 \end{pmatrix}$

e) $v = \left(1, 0, \frac{1}{3}\right)^T, A = \begin{pmatrix} -57 & 120 & 180 \\ -18 & 39 & 54 \\ -6 & 12 & 21 \end{pmatrix}$ f) $v = \left(1, 0, -\frac{1}{2}\right)^T, A = \begin{pmatrix} 10 & 0 & 14 \\ 14 & 3 & 28 \\ -7 & 0 & -11 \end{pmatrix}$

3. 已知矩阵 A 的一个特征值为 λ，求它所对应的特征向量。其中 λ, A 为

a) $\lambda=5, A=\begin{pmatrix} 3 & 0 \\ -2 & 5 \end{pmatrix}$

b) $\lambda=0, A=\begin{pmatrix} 0 & 0 \\ -2 & -2 \end{pmatrix}$

c) $\lambda=2, A=\begin{pmatrix} 4 & -8 & -2 \\ 1 & -2 & -1 \\ -1 & 4 & 3 \end{pmatrix}$

d) $\lambda=1, A=\begin{pmatrix} 33 & -64 & 96 \\ 12 & -23 & 36 \\ -4 & 8 & -11 \end{pmatrix}$

e) $\lambda=-2, A=\begin{pmatrix} 8 & 10 & 30 \\ -20 & -22 & -60 \\ 5 & 5 & 13 \end{pmatrix}$

f) $\lambda=4, A=\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ -6 & -12 & -2 \end{pmatrix}$

4. 求矩阵的特征值与特征向量

a) $A=\begin{pmatrix} 6 & 48 \\ -4 & -22 \end{pmatrix}$

b) $A=\begin{pmatrix} 9 & 0 \\ -13 & -4 \end{pmatrix}$

c) $A=\begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$

d) $A=\begin{pmatrix} -8 & 0 \\ -8 & 0 \end{pmatrix}$

e) $A=\begin{pmatrix} 8 & 11 & -22 \\ -36 & -25 & 30 \\ -18 & -11 & 12 \end{pmatrix}$

f) $A=\begin{pmatrix} -1 & 1 & 3 \\ 12 & 1 & 18 \\ -4 & -1 & -8 \end{pmatrix}$

g) $A=\begin{pmatrix} 48 & 4 & 152 \\ -13 & 7 & -50 \\ -13 & -1 & -42 \end{pmatrix}$

h) $A=\begin{pmatrix} 24 & 8 & 64 \\ 64 & 20 & 176 \\ -16 & -4 & -40 \end{pmatrix}$

i) $A=\begin{pmatrix} -52 & -240 & 0 \\ 12 & 56 & 0 \\ -12 & -48 & 8 \end{pmatrix}$

j) $A=\begin{pmatrix} 37 & 0 & -160 \\ -16 & -3 & 64 \\ 8 & 0 & -35 \end{pmatrix}$

答案

1. a) $r^2 - r - 7$
   b) $r^2 - 3r + 27$
   c) $r^3 + 10r^2 + 7r - 6$
   d) $r^3 - 8r^2 + 29r + 58$
   e) $r^3 + 4r^2 - 18r - 164$
   f) $r^3 + r^2 - 6r - 116$

2. a) 0
   b) 2
   c) -2
   d) 0
   e) -3
   f) -4

3. a) $(0,\,1)^T$
   b) $(1,\,-1)^T$
   c) $v_1 = (1,\,0,\,1)^T, v_2 = (0,\,1,\,-4)^T$
   d) $v_1 = \left(1,\,0,\,-\frac{1}{3}\right)^T, v_2 = \left(0,\,1,\,\frac{2}{3}\right)^T$
   e) $v_1 = \left(1,\,0,\,-\frac{1}{3}\right)^T, v_2 = \left(0,\,1,\,-\frac{1}{3}\right)^T$
   f) $v_1 = (1,\,0,\,-1)^T, v_2 = (0,\,1,\,-2)^T$

4. a) $\lambda_1 = -6, v_1 = \left(1,\,-\frac{1}{4}\right)^T, \quad \lambda_2 = -10, v_2 = \left(1,\,-\frac{1}{3}\right)^T$
   b) $\lambda_1 = 9, v_1 = (1,\,-1)^T, \quad \lambda_2 = -4, v_2 = (0,\,1)^T$
   c) $\lambda_1 = 9, v_1 = (1,\,0)^T, \quad \lambda_2 = 9, v_2 = (0,\,1)^T$
   d) $\lambda_1 = 0, v_1 = (0,\,1)^T, \quad \lambda_2 = -8, v_2 = (1,\,1)^T$
   e) $\lambda_1 = 8, v_1 = (1,\,-2,\,-1)^T, \lambda_2 = -3, v_2 = (1,\,-3,\,-1)^T, \lambda_3 = -10, v_3 = \left(0,\,1,\,\frac{1}{2}\right)^T$
   f) $\lambda_1 = -1, v_1 = (1,\,3,\,-1)^T, \lambda_2 = -2, v_2 = (1,\,2,\,-1)^T, \lambda_3 = -5, v_3 = \left(0,\,1,\,-\frac{1}{3}\right)^T$
   g) $\lambda_1 = 9, v_1 = \left(1,\,-\frac{1}{4},\,-\frac{1}{4}\right)^T, \lambda_2 = 8, v_2 = \left(1,\,-\frac{1}{2},\,-\frac{1}{4}\right)^T, \lambda_3 = -4, v_3 = \left(1,\,-\frac{1}{3},\,-\frac{1}{3}\right)^T$
   h) $\lambda_1 = 8, v_1 = \left(1,\,2,\,-\frac{1}{2}\right)^T, \lambda_2 = 4, v_2 = \left(1,\,\frac{3}{2},\,-\frac{1}{2}\right)^T, \lambda_3 = -8, v_3 = (1,\,4,\,-1)^T$
   i) $\lambda_1 = -4, v_1 = \left(1,\,-\frac{1}{5},\,\frac{1}{5}\right)^T, \lambda_2 = 8, v_2 = \left(1,\,-\frac{1}{4},\,0\right)^T, \lambda_3 = 8, v_3 = (0,\,0,\,1)^T$
   j) $\lambda_1 = 5, v_1 = \left(1,\,-\frac{2}{5},\,\frac{1}{5}\right)^T, \lambda_2 = -3, v_2 = \left(1,\,0,\,\frac{1}{4}\right)^T, \lambda_3 = -3, v_3 = (0,\,1,\,0)^T$