特征值与特征向量

练习

1. 求矩阵 A 的特征多项式：

a) $\begin{pmatrix} -5 & -3 \\ 2 & 5 \end{pmatrix}$

b) $\begin{pmatrix} -3 & -1 \\ -2 & -2 \end{pmatrix}$

c) $\begin{pmatrix} 2 & -4 & -4 \\ 0 & 4 & -1 \\ -3 & -5 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 4 & -1 \\ -2 & 3 & -3 \\ -4 & 4 & -5 \end{pmatrix}$

e) $\begin{pmatrix} 3 & 5 & 5 \\ -4 & 5 & 4 \\ -4 & -1 & -4 \end{pmatrix}$

f) $\begin{pmatrix} 0 & -1 & -5 \\ 4 & -2 & 5 \\ -2 & 1 & 5 \end{pmatrix}$

2. 已知矩阵 A 的一个特征向量为 $\mathbf{v}$，求它所对应的特征值。其中 $A, \mathbf{v}$ 为

a) $v = \left(1,\, -\dfrac{1}{4}\right)^T, A = \begin{pmatrix} 32 & 108 \\ -9 & -31 \end{pmatrix}$

b) $v = \left(1,\, \dfrac{1}{4}\right)^T, A = \begin{pmatrix} -45 & 200 \\ -10 & 45 \end{pmatrix}$

c) $v = \left(1,\, 0,\, -\dfrac{1}{4}\right)^T, A = \begin{pmatrix} 8 & 0 & 12 \\ 1 & 5 & 4 \\ -1 & 0 & 1 \end{pmatrix}$

d) $v = (0,\, 1,\, 0)^T, A = \begin{pmatrix} -1 & 0 & 0 \\ -8 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

e) $v = \left(1,\, 0,\, \dfrac{1}{2}\right)^T, A = \begin{pmatrix} 20 & -54 & -36 \\ 12 & -34 & -24 \\ -6 & 18 & 14 \end{pmatrix}$

f) $v = \left(1,\, 0,\, -\dfrac{1}{2}\right)^T, A = \begin{pmatrix} 15 & 36 & 36 \\ -8 & -19 & -16 \\ -2 & -4 & -7 \end{pmatrix}$

3. 已知矩阵 A 的一个特征值为 λ，求它所对应的特征向量。其中 λ, A 为

a) $\lambda = 2, A = \begin{pmatrix} -23 & 100 \\ -5 & 22 \end{pmatrix}$

b) $\lambda = 5, A = \begin{pmatrix} -19 & 48 \\ -8 & 21 \end{pmatrix}$

c) $\lambda = -1, A = \begin{pmatrix} 47 & 144 & 192 \\ -12 & -37 & -48 \\ -4 & -12 & -17 \end{pmatrix}$

d) $\lambda = -3, A = \begin{pmatrix} -7 & 0 & 12 \\ 1 & -3 & -3 \\ -1 & 0 & 0 \end{pmatrix}$

e) $\lambda = 0, A = \begin{pmatrix} -11 & 44 & -44 \\ -2 & 8 & -8 \\ 1 & -4 & 4 \end{pmatrix}$

f) $\lambda = 1, A = \begin{pmatrix} 1 & 0 & 0 \\ 15 & 16 & 60 \\ -5 & -5 & -19 \end{pmatrix}$

4. 求矩阵的特征值与特征向量

a) $A = \begin{pmatrix} 5 & 0 \\ -13 & -8 \end{pmatrix}$

b) $A = \begin{pmatrix} 15 & 36 \\ -6 & -15 \end{pmatrix}$

c) $A = \begin{pmatrix} -7 & 6 \\ -1 & -12 \end{pmatrix}$

d) $A = \begin{pmatrix} 26 & 72 \\ -12 & -34 \end{pmatrix}$

e) $A = \begin{pmatrix} -4 & 0 & 0 \\ -12 & 2 & 6 \\ -12 & -3 & 11 \end{pmatrix}$

f) $A = \begin{pmatrix} 39 & 72 & -108 \\ -6 & -9 & 18 \\ 6 & 12 & -15 \end{pmatrix}$

g) $A = \begin{pmatrix} -16 & -2 & 42 \\ -18 & -2 & 48 \\ -6 & -1 & 17 \end{pmatrix}$

h) $A = \begin{pmatrix} 3 & 4 & 4 \\ 11 & 3 & 11 \\ -11 & -4 & -12 \end{pmatrix}$

i) $A = \begin{pmatrix} -45 & -32 & 232 \\ -10 & -11 & 56 \\ -10 & -8 & 53 \end{pmatrix}$

j) $A = \begin{pmatrix} 21 & 12 & 18 \\ -15 & -6 & -18 \\ -15 & -6 & -18 \end{pmatrix}$

答案

1. a) r^2-19 b) r^2+5r+4
   c) $r^3-7r^2-3r+62$ d) r^3+2r^2+r-4
   e) $r^3-4r^2+27r+88$ f) $r^3-3r^2-21r-30$

2. a) 5 b) 5
   c) 4 d) -1
   e) -4 f) -5

3. a) $\left(1,\ \frac{1}{4}\right)^T$ b) $\left(1,\ \frac{1}{2}\right)^T$
   c) $v_1=\left(1,\ 0,\ -\frac{1}{4}\right)^T, v_2=\left(0,\ 1,\ -\frac{3}{4}\right)^T$ d) $v_1=\left(1,\ 0,\ \frac{1}{3}\right)^T, v_2=(0,\ 1,\ 0)^T$
   e) $v_1=\left(1,\ 0,\ -\frac{1}{4}\right)^T, v_2=(0,\ 1,\ 1)^T$ f) $v_1=\left(1,\ 0,\ -\frac{1}{4}\right)^T, v_2=\left(0,\ 1,\ -\frac{1}{4}\right)^T$

4. a) $\lambda_1=5, v_1=(1,\ -1)^T, \quad \lambda_2=-8, v_2=(0,\ 1)^T$
   b) $\lambda_1=3, v_1=\left(1,\ -\frac{1}{3}\right)^T, \quad \lambda_2=-3, v_2=\left(1,\ -\frac{1}{2}\right)^T$
   c) $\lambda_1=-9, v_1=\left(1,\ -\frac{1}{3}\right)^T, \quad \lambda_2=-10, v_2=\left(1,\ -\frac{1}{2}\right)^T$
   d) $\lambda_1=2, v_1=\left(1,\ -\frac{1}{3}\right)^T, \quad \lambda_2=-10, v_2=\left(1,\ -\frac{1}{2}\right)^T$
   e) $\lambda_1=8, v_1=(0,\ 1,\ 1)^T, \lambda_2=5, v_2=\left(0,\ 1,\ \frac{1}{2}\right)^T, \lambda_3=-4, v_3=(1,\ 1,\ 1)^T$
   f) $\lambda_1=9, v_1=\left(1,\ -\frac{1}{6},\ \frac{1}{6}\right)^T, \lambda_2=3, v_2=\left(1,\ 0,\ \frac{1}{3}\right)^T, \lambda_3=3, v_3=\left(0,\ 1,\ \frac{2}{3}\right)^T$
   g) $\lambda_1=2, v_1=\left(1,\ \frac{3}{2},\ \frac{1}{2}\right)^T, \lambda_2=1, v_2=\left(1,\ 2,\ \frac{1}{2}\right)^T, \lambda_3=-4, v_3=\left(1,\ 1,\ \frac{1}{3}\right)^T$
   h) $\lambda_1=3, v_1=(1,\ 1,\ -1)^T, \lambda_2=-1, v_2=(1,\ 0,\ -1)^T, \lambda_3=-8, v_3=(0,\ 1,\ -1)^T$
   i) $\lambda_1=5, v_1=\left(1,\ \frac{1}{4},\ \frac{1}{4}\right)^T, \lambda_2=-3, v_2=\left(1,\ \frac{1}{2},\ \frac{1}{4}\right)^T, \lambda_3=-5, v_3=\left(1,\ \frac{1}{5},\ \frac{1}{5}\right)^T$
   j) $\lambda_1=6, v_1=\left(1,\ -\frac{1}{2},\ -\frac{1}{2}\right)^T, \lambda_2=0, v_2=\left(1,\ -1,\ -\frac{1}{2}\right)^T, \lambda_3=-9, v_3=(1,\ -1,\ -1)^T$