

定积分的定义与计算

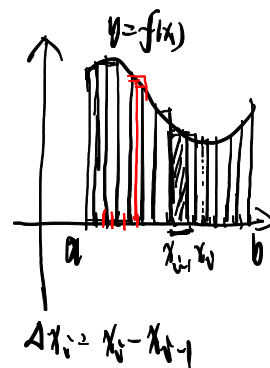
1. 定积分的定义

$$\Delta A_i \approx \underline{f(x_i^*)} \Delta x_i \quad x_i^* \in (x_{i-1}, x_i)$$

$$A = \sum_{i=1}^n \Delta A_i \approx \sum_{i=1}^n \underline{f(x_i^*)} \Delta x_i$$

$$\Delta x \rightarrow 0 \quad \Delta x = \max \{\Delta x_i\}$$

$$\Rightarrow A = \lim_{\substack{\Delta x \rightarrow 0 \\ n \rightarrow \infty}} \sum_{i=1}^n \underline{f(x_i^*)} \Delta x_i = \int_a^b f(x) dx$$



2. 计算

定理: $F(x)$ 是 $f(x)$ 的原函数 $\Rightarrow \int_a^b f(x) dx = F(b) - F(a)$ (牛顿-莱布尼兹)

(微积分基本定理)

$\Rightarrow$ 不定积分

3. 特殊计算技巧 (方法) (换元必换限)

① 换元必换限

$$\begin{aligned} \int_0^4 \frac{x+2}{\sqrt{x+1}} dx & \quad \begin{array}{l} u = \sqrt{x+1} \\ x = u^2 - 1 \\ dx = 2u du \end{array} \quad \int_1^3 \frac{\frac{1}{2}(u^2-1)+2}{u} \cdot 2u du = \int_1^3 \left(\frac{1}{2}u^2 + \frac{3}{2} \right) du \\ & = \left(\frac{1}{6}u^3 + \frac{3}{2}u \right) \Big|_1^3 = \frac{1}{6} \cdot 27 + \frac{3}{2} \cdot 3 - \frac{1}{6} - \frac{3}{2} \\ & = \frac{9}{2} + \frac{9}{2} - \frac{10}{6} = 9 - \frac{5}{3} = \frac{22}{3} \end{aligned}$$

② 奇函数在对称区间上积分为 0, $f(x)$ 奇函数.

$$\int_{-a}^a f(x) dx = 0$$

$$\Rightarrow \int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

$$\stackrel{t=-x}{=} \int_a^0 f(-t) \cdot (-dt) + \int_0^a f(x) dx$$

$$= \int_0^a f(-t) dt + \int_0^a f(x) dx$$

$$= - \int_0^a f(t) dt + \int_0^a f(x) dx = 0$$

$$\int_{-2}^2 \frac{\sin x}{x^2 + \cos x} dx = 0$$

③ 注意此题的值

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^2 x - \cos^4 x} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^2 x (1 - \cos^2 x)} dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \cdot |\sin x| dx$$

$$= \int_{-\frac{\pi}{2}}^0 \cos x \cdot (-\sin x) dx + \int_0^{\frac{\pi}{2}} \cos x \cdot \sin x dx$$