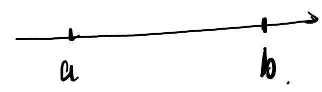
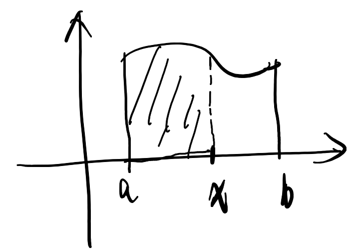
# 变上限积分及其导数

1. $S'(t) = V(t)$




$$\int_a^b v(t)\,dt = S(b) - S(a)$$

2. 变上限积分、若 $f(x)$ 在 $[a,b]$ 上连续，定义




$$F(x) = \int_a^x f(t)\,dt$$

定理：若 $f(x)$ 在 $[a,b]$ 上连续，那么　　（微积分基本定理 I）

$$F(x) = \int_a^x f(t)\,dt$$

在 $[a,b]$ 上连续，在 $(a,b)$ 内可导，且 $F'(x) = f(x)$　（$F(x)$ 是 $f(x)$ 的一个原函数）

证明：$F(x+h) - F(x) = \int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt$

$$= \int_x^{x+h} f(t)\,dt = f(\xi)\cdot h \qquad \xi \in (x, x+h)$$

$h \to 0 \Rightarrow F(x+h) - F(x) \to 0 \Rightarrow F(x+h) \to F(x) \quad (h \to 0)$

i.e. $\lim\limits_{h\to 0} F(x+h) = F(x) \Rightarrow F(x)$ 连续.

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \frac{f(\xi)\cdot h}{h} = \lim_{h\to 0} f(\xi) = f(x).$$

例1. 求 $F(x) = \int_0^x \sqrt{1+t^2}\,dt$ 的导数.

解：$F'(x) = f(x) = \sqrt{1+x^2}$

例2. 设 $F(x) = \int_1^{x^4} \sec t\,dt$，求 $F'(x)$

解：设 $F(u) = \int_1^u \sec t\,dt$，$u = x^4 \Rightarrow F'(x) = F'(u)\cdot \frac{du}{dx}$

$$= \sec u \cdot 4x^3$$
$$= \sec(x^4)\cdot 4x^3$$

3. 一般地. ① $F(x)=\int_a^x f(t)dt$, $F'(x)=f(x)$

② $F(x)=\int_a^{\varphi(x)} f(t)dt$, $F'(x)=f(\varphi(x))\cdot\varphi'(x)$

③ $F(x)=\int_{\varphi(x)}^{\psi(x)} f(t)dt$, $F'(x)=f(\psi(x))\psi'(x)-f(\varphi(x))\cdot\varphi'(x)$

证明. $F(x)=\int_{\underline{\underline{\varphi(x)}}}^{a} f(t)dt+\int_a^{\psi(x)} f(t)dt$

$=\int_a^{\psi(x)} f(t)dt-\int_a^{\varphi(x)} f(t)dt$

然后应用②.

补充: 定积分: ① $\int_a^b f(x)dx=0$

② $\int_a^b f(x)dx=-\int_b^a f(x)dx$ ($\because \Delta x\le 0$)