

交换积分次序

① 画图

② 找出积分上下限

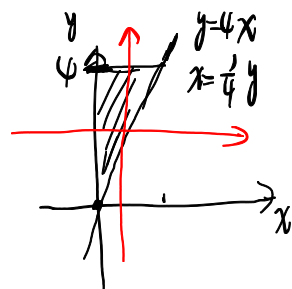
例1. $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$

解: 积分区域如图

$$D: \{(x, y) \mid 0 \leq x \leq \frac{1}{4}y, 0 \leq y \leq 4\}$$

$$\begin{aligned} \Rightarrow \int_0^1 \int_{4x}^4 e^{-y^2} dy dx &= \int_0^4 \int_0^{\frac{1}{4}y} e^{-y^2} dx dy \\ &= \int_0^4 e^{-y^2} x \Big|_0^{\frac{1}{4}y} dy = \int_0^4 \frac{1}{4}y e^{-y^2} dy \end{aligned}$$

$$\begin{aligned} &\stackrel{u=y^2}{=} -\frac{1}{8} \int_0^4 e^{-u} du = -\frac{1}{8} e^{-u} \Big|_0^4 = -\frac{1}{8}(e^{-4} - 1) \\ &= \frac{1}{8}(1 - e^{-4}) \end{aligned}$$



例2. $\int_0^2 \int_{\frac{y}{2}}^1 \cos(x^2) dx dy$

解: 积分区域如图

$$D: \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2x\}$$

$$\begin{aligned} \Rightarrow \int_0^2 \int_{\frac{y}{2}}^1 \cos(x^2) dx dy &= \int_0^1 \int_0^{2x} \cos(x^2) dy dx \\ &= \int_0^1 y \cos(x^2) \Big|_0^{2x} dx = \int_0^1 2x \cos(x^2) dx \\ &= \sin x^2 \Big|_0^1 = \sin 1 \end{aligned}$$

